SNUTP10-004
arXiv:1003.4343

Probing AdS_4/CFT_3 proposals beyond chiral rings

Seok Kim[1] and Jaemo Park[2]

[1] *Department of Physics and Astronomy & Center for Theoretical Physics, Seoul National University, Seoul 151-747, Korea.*

[2] *Department of Physics & Postech Center for Theoretical Physics (PCTP), POSTECH, Pohang 790-784, Korea*

E-mails: `skim@phya.snu.ac.kr`, `jaemo@postech.ac.kr`

Abstract

We calculate the superconformal Witten index for the Chern-Simons-matter theory which was proposed to describe multiple M2-branes on $\mathbb{C}^2 \times \mathbb{C}^2/\mathbb{Z}_k$. We consider a variant of this model, which exhibits explicit $\mathcal{N}=3$ supersymmetry and has the advantage of not having an exotic branch of the moduli space. At $k=1$, we compare the index with that from the proposed gravity dual and find a disagreement.

Contents

1 Introduction **1**

2 Dual ABJM and its variant **3**

3 The index for the dual ABJM **5**

1 Introduction

Recently, we have seen the tremendous progress in understanding AdS_4/CFT_3 correspondence. The gravitational part involving AdS_4 was relatively well-known. The harder problem was to understand CFT_3 corresponding to M2 branes probing a part of Calabi-Yau 4-fold. The crucial observation was made by Schwarz [1] that these CFT_3s can be described by Chern-Simons gauge theory with matter without the usual Yang-Mills kinetic terms. In retrospect, the $\mathcal{N}=8$ theory of Bagger-Lambert-Gustavsson [2, 3, 4, 5, 6] is the avatar of this idea, which is equivalent to $SU(2)\times SU(2)$ gauge theory [7]. More general classes of $\mathcal{N}=4$ Chern-Simons theories describing M2 brane probing suitable non-compact Calabi-Yau manifold were constructed by [8] following the method of [9]. It turns out that the special case of the construction of [8] describes M2 branes probing C^4/Z_k, whose AdS/CFT aspects were worked out in detail in [10], which is called ABJM theory with enhanced $\mathcal{N}=6$ supersymmetry [11, 12].

The analysis of $\mathcal{N}=2$ theories were initiated in [13, 14]. This is an interesting arena to work out since $\mathcal{N}=2$ theories correspond to M2 branes probing general Calabi-Yau four-folds. The curious fact of $\mathcal{N}=2$ theory is that there could be multiple Chern-Simons gauge theories having the same moduli space. The typical example is the so-called the dual ABJM theory which is $U(N)\times U(N)$ $\mathcal{N}=2$ theory with bifundamentals $A\,(N,\bar{N})$, $B\,(\bar{N},N)$ and with additional adjoint fields ϕ_1,ϕ_2 of the first gauge group factor. With the superpotential $W=\mathrm{Tr}AB[\phi_1,\phi_2]$ and with the Chern-Simons level $(k,-k)$, one can show that the moduli space contains the symmetric product of $C^2/Z_k\times C^2$.[1] For $k=1$ the moduli space has the symmetric product of C^4. One might suspect that the ABJM and the dual ABJM theory could have the identical gravitational dual for $k=1$. On the other hand, it is observed that there are at least 19 models of $\mathcal{N}=2$ theory which has the symmetric product of C^4 as the moduli space [17]. Thus the important question is if all of these models are describing M2 branes probing C^4. Since all of these theories involve the Chern-Simons level $k=0,1$, it's rather difficult to analyze these theories perturbatively. There'a a possibility that some of these theories might

[1]Another model with moduli space of $C^2/Z_k\times C^2$ is constructed by using Chern-Simons matter theories with chiral flavors [15]. Also in [16] dual ABJM model is obtained via orbifold procedure of Nambu bracket.

be related via Seiberg-duality.

In order to answer this question, we should work out more than the chiral ring or moduli space of the underlying theories. Recently there were several attempts along this direction. One is the compuatation of the superconformal Witten index [18, 19], which provides the detailed information of BPS states (or local operators) with lower supersymmetry. Another is the computation of the partition function and the Wilson loop using the localization technique [20, 21]. Especially in [22], 1/2 BPS Wilson loop is evaluated exactly in ABJM theory. The goal of this letter is to compare the indices of two theories which have the same moduli space. The theories we are interested in are ABJM and $\mathcal{N}=3$ variant of the dual ABJM theory which we introduce in section 2. The moduli space of the $\mathcal{N}=3$ variant, with Chern-Simons level $(k,-k)$, is given by the symmetric product of $C^2/Z_k \times C^2$. Furthermore it does not have additional exotic branches, in contrast to $\mathcal{N}=2$ dual ABJM model [23]. Besides, we believe that the $\mathcal{N}=3$ theory is better suited for the index computation for technical reasons. One can trust the various assumptions made in the index computation while for $\mathcal{N}=2$ theories, this does not seem to be the case, on which we will comment later. In particular, one can carry out the index computation for Chern-Simons level $(k,-k)=(1,-1)$ where the underlying moduli space is the symmetric product of C^4. One can see that the index computation of $\mathcal{N}=3$ theory leads to the different result than ABJM case with $(k,-k)=(1,-1)$. Thus one can show explicitly that having the same moduli space does not necessarily give the same M2 brane CFT. Given this result, one might wonder what is the corresponding gravitational dual of the $\mathcal{N}=3$ theory or whether this $\mathcal{N}=3$ theory has the gravitational dual at all. We will also explain a deconfinement behavior of the index, which seems to be in tension with the M2 brane interpretation.

One may ask similar questions for various $\mathcal{N}=2$ theories having the symmetric product of C^4 as the moduli space, though we cannot show any computation for these theories. One interesting example related to this question is again found in $\mathcal{N}=6$ theory. If we consider $\mathcal{N}=6$ $U(N+l)_k \times U(N)_{-k}$ theory with $l \leq k$, they have the same moduli space $Sym^N(C^4/Z_k)$ but they have different gravitational duals distinguished by the different discrete flux data in the gravitational side [24]. We hope to understand if various $\mathcal{N}=2$ theories are related in a similar way or another, but this is beyond the scope of the current work. Finanlly, the current work has an interesting implication for the relation between the crystal models and the Chern-Simons matter theories in (2+1)-dimensions [25, 26, 27]. There are various proposals for extracting the gauge theory from the crystal models[27, 29]. Especially in [29], it is suggested that ABJM and dual ABJM model are two theories read off from C^4 crystal model. Given our result, this proposal should be modified. And it is an interesting topic to figure out the precise gauge theory associated with a given crytal model, given a successful construction of various $\mathcal{N}=2$ theories where the corresponding crystal models [30] play a crucial role.

2 Dual ABJM and its variant

A (2+1)d $\mathcal{N}=2$ Chern-Simons(CS) theory with bifundamental and adjoint matter is given, in $\mathcal{N}=2$ superspace notation, by the following Lagrangian [14]

$$\mathrm{Tr}\left(-\int d^4\theta \sum_{X_{ab}} X_{ab}^\dagger e^{-V_a} X_{ab} e^{V_b} - \sum_a \frac{k_a}{2\pi}\int_0^1 dt V_a \bar{D}^\alpha (e^{tV_a} D_\alpha e^{-tV_a}) + \int d^2\theta W(X_{ab}) + \text{c.c.}\right) , \tag{2.1}$$

where V_a are vector supermultiplets and X_{ab} denote chiral supermultiplets transforming in the fundamental representation of gauge group a and the anti-fundamental representation of gauge group b. For $a=b$, this corresponds to adjoint matter for gauge group a. We take $\sum k_a = 0$. This is a necessary condition for the moduli space to be four complex dimensional. Recall that in 2+1 dimensions a vector superfield has the expansion

$$V = -2i\theta\bar{\theta}\sigma + 2\theta\gamma^\mu\bar{\theta}A_\mu + \cdots + \theta^2\bar{\theta}^2 D , \tag{2.2}$$

where we omitted the fermionic part. Compared to (3+1)-dimensions, there is a new scalar field σ. We can write all terms contributing to the scalar potential in the Lagrangian

$$\mathrm{Tr}\left(-\sum_a \frac{2k_a}{\pi}\sigma_a D_a + \sum_a D_a \mu_a(X) - \sum_{X_{ab}} (\sigma_a X_{ab} - X_{ab}\sigma_b)(\sigma_a X_{ab} - X_{ab}\sigma_b)^\dagger - \sum_{X_{ab}} |\partial_{X_{ab}} W|^2\right) . \tag{2.3}$$

$\mu_a(X)$ is the moment map for the a-th gauge group

$$\mu_a(X) = \sum_b X_{ab}X_{ab}^\dagger - \sum_c X_{ca}^\dagger X_{ca} + [X_{aa}, X_{aa}^\dagger] , \tag{2.4}$$

and gives the D-term. Here we use the same terminology of (3+1)d.

By integrating out the auxiliary fields D_a, we see that the bosonic potential is a sum of squares. The vacua can be found by looking for vanishing of the scalar potential. This gives rise to a set of matrix equations

$$\begin{aligned} \partial_{X_{ab}} W &= 0 \\ \mu_a(X) &= \frac{2k}{\pi} k_a \sigma_a \\ \sigma_a X_{ab} - X_{ab}\sigma_b &= 0 \end{aligned} \tag{2.5}$$

The solutions to these equations automatically satisfy $D_a = 0$ and correspond to supersymmetric vacua. F-term constraints are exactly as in the (3+1)d case, while D-term constraints are modified.

In this letter, we consider a special class of $\mathcal{N}=2$ theory where the gauge group is given by $U(N_1)\times U(N_2)$ with two bifundamentals $A(N_1,\bar{N}_2)$ and $B(\bar{N}_1, N_2)$ and with two adjoints ϕ_1, ϕ_2

of $U(N_1)$. Chern-Simons level is given by $(k, -k)$. We introduce the superpotential for this theory to obtain $\mathcal{N}=3$ theory. Following [28], we introduce Φ_1, Φ_2 auxiliary chiral superfields of the adjoint representation of $U(N_1), U(N_2)$ respectively. Combined with V_a of $\mathcal{N}=2$ vector superfield, they form $\mathcal{N}=4$ vectormultiplets. $\mathcal{N}=3$ superpotential is obtained by starting from

$$\int d^2\theta(-\frac{k}{4\pi}\mathrm{Tr}\Phi_1^2 + \mathrm{Tr}\Phi_1[\phi_1,\phi_2] + B\Phi_1 A + \frac{k}{4\pi}\mathrm{Tr}\Phi_2^2 + A\Phi_2 B) \tag{2.6}$$

and integrating out Φ_1, Φ_2 so that

$$W = \frac{2\pi}{k}\mathrm{Tr}\left(2AB[\phi_1,\phi_2] + [\phi_1,\phi_2]^2\right) \ . \tag{2.7}$$

Note that the $\mathcal{N}=2$ dual ABJM is given by the superpotential [29, 30]

$$W = \frac{4\pi}{k}\mathrm{Tr}AB[\phi_1,\phi_2]. \tag{2.8}$$

Assume that $N_1 = N_2 \equiv N$. Let's work out the moduli space of abelian case of $\mathcal{N}=3$ case. The superpotential is vanishing identically and we have $\sigma_1 = -\sigma_2 \equiv \sigma$ with

$$\mu_1 = -\mu_2 = \frac{2k}{\pi}\sigma. \tag{2.9}$$

Thus the moment map determines the value of σ_a. By imposing the integer quantization condition of the flux associated with $F = dA_1 + dA_2$ where A_1, A_2 are the gauge fields of two U(1)s, of $\frac{1}{k}$, one fixes the periodicity of the conjugate variable of $A_1 + A_2$, which gives the usual discrete modding[31]

$$A \to e^{\frac{2\pi i}{k}}A \quad B \to e^{-\frac{2\pi i}{k}}B \tag{2.10}$$

while ϕ_i is left invariant. Thus we have the moduli space $C^2/Z_k \times C^2$. Note that the moduli space is identical to the dual ABJM for the abelian case. For nonabelian case, both $\mathcal{N}=3$ theory and the $\mathcal{N}=2$ dual ABJM model have the symmetric product of $C^2/Z_k \times C^2$ as one branch of the moduli space. The important question is if this is the only branch of the moduli space. In [23], it is shown that $\mathcal{N}=2$ dual ABJM model has the additional branch of the moduli space. Below, we show that the $\mathcal{N}=3$ theory does not have such additional branch so that its moduli space is simply given by the symmetric product of $C^2/Z_k \times C^2$.

We first consider the F-term condition for ϕ_1, ϕ_2, $M \equiv AB$:

$$[\phi_1,\phi_2]M = M[\phi_1,\phi_2] = 0 \ , \tag{2.11}$$

$$[\phi_i, M + [\phi_1,\phi_2]] = 0 \quad (i=1,2) \ . \tag{2.12}$$

M is in the adjoint representation of the first $U(N)$ gauge group. Compared to the case studied in [23], we have $[\phi_1,\phi_2]$ inside the commutator of (2.12). Subtracting the two equations in (2.11), we can use $GL(N)$ which complexifies first $U(N)$ to diagonalize both M and $[\phi_1,\phi_2]$. Let the two matrices have eigenvalues m_I and λ_I, respectively, with $I = 1, 2, \cdots, N$. The conditions

in (2.11) imply $m_I \lambda_I = 0$, so one of the two eigenvalues for given I is always zero. Now we consider the last condition (2.12). In the above diagonalizing basis, $M+[\phi_1, \phi_2]$ is also diagonal. (2.12) implies that ϕ_i can be 'generically' diagonalized in the same basis.

If there turns out to be a block of equal eigenvalues in $M+[\phi_1, \phi_2]$, (2.12) does not constrain off-diagonal entries of ϕ_i in this block. Since either eigenvalues of m_I or λ_I is zero, equal eigenvalues $m_I+\lambda_I$ of $M+[\phi_1, \phi_2]$ for different I may appear in following possibilities. Firstly, some λ_I's can be zero while corresponding m_I's are all equal. In this block, $[\phi_1, \phi_2] = 0$ from $\lambda_I = 0$ and ϕ_1, ϕ_2 can clearly be diagonalized simultaneously. Secondly, some m_I's can be zero while corresponding λ_I's are all equal. In this block, we have $M = 0$ so that part of $GL(N)$ is not used, while $[\phi_1, \phi_2]$ is unconstrained from (2.11). We use $GL(N)$ to diagonalize ϕ_1 with eigenvalues α_I. Inserting $M=0$ to (2.12) with $i=1$, one obtains

$$[\phi_1, [\phi_1, \phi_2]] = 0 \ . \tag{2.13}$$

In this block, the IJ'th element is given by

$$(\alpha_I - \alpha_J)^2 (\phi_2)_{IJ} = 0 \tag{2.14}$$

so that the off-diagonals of ϕ_2 is either zero or ϕ_2 can be diagonalized if some eigenvalues of ϕ_1 are equal. Finally, one would worry about the blocks in which $m_I = \lambda_J \neq 0$ with $m_J = 0 = \lambda_I$ (for $I \neq J$), so that the eigenvalues of $M + [\phi_1, \phi_2]$ are still equal in that block. For the two sub-blocks with nonzero M and zero M, we have diagonalized the fields and all that matter are off-diagonal elements $(\phi_i)_{IJ}$ with $m_I = \lambda_J$, $m_J = 0 = \lambda_I$ (for $I \neq J$). However, we already know that $\lambda_J = 0$ since ϕ_i are all diagonalized by eq. (2.13) and (2.14), so the last case actually does not exist.

Having diagonalized M, ϕ_1, ϕ_2 using the first $U(N)$ gauge group, one can use the second $U(N)$ to diagonalize A, B separately.

Therefore, unlike the $\mathcal{N}=2$ proposal for dual ABJM, our $\mathcal{N}=3$ version does not have an exotic branch in the moduli space in which $M = 0$ and ϕ_1, ϕ_2 do not commute [23]. The crucial difference is (2.12): without $[\phi_1, \phi_2]$ inside the commutator, $M = 0$ will trivialize this condition while not in our case.

3 The index for the dual ABJM

The dual ABJM model has four global $U(1)$ symmetries, which we parametrize as h_1, h_2, h_3, h_4. Our notation is such that h_3, h_4 form the Cartans of $SO(4)$ R-symmetry, had there been such an enhancement somehow.[2] Our BPS relation is $\epsilon = h_3 + j_3$, and the global $U(1)_b$ charge whose associated gauge non-invariance has to be screened by monopole operators is $\frac{h_1+h_2}{2}$. The BPS

[2]They are denoted by h_1, h_2 in [32].

fields	$U(N)\times U(N)$	h_1	h_2	h_3	h_4	j_3	ϵ
A	$(N,\bar{N})$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$
B	$(\bar{N},N)$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$
$\psi_\pm$	$(N,\bar{N})$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\pm\frac{1}{2}$	1
$\chi_\pm$	$(\bar{N},N)$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\pm\frac{1}{2}$	1
ϕ_1	$(\text{adj},1)$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$
ϕ_2	$(\text{adj},1)$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$
$\Psi_{1\pm}$	$(\text{adj},1)$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\pm\frac{1}{2}$	1
$\Psi_{2\pm}$	$(\text{adj},1)$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\pm\frac{1}{2}$	1
$A_\mu,\tilde{A}_\mu$		0	0	0	0	$(1,0,-1)$	1

Table 1: charges of fields in dual ABJM

fields in the free field theory are $\bar{A}$, $\bar{B}$, $\bar{\phi}_i$, ψ_+, χ_+, Ψ_{i+} and derivatives D_{++}. The index over modes (or letters) is defined as

$$f_R(x,y_1,y_2)=\text{tr}\left[(-1)^F e^{-\beta'\{Q,S\}}x^{\epsilon+j_3}y_1^{\frac{h_1-h_2}{2}}y_2^{h_4}\right]\ , \tag{3.1}$$

where the trace is taken over the normal modes (not over the full space of local gauge invariant operators.) The letter indices in bi-fundamental, anti-bi-fundamental and two adjoint representations in the free theory are given by

$$f_+=f_-=\frac{x^{\frac{1}{2}}y_2^{-\frac{1}{2}}-x^{\frac{3}{2}}y_2^{\frac{1}{2}}}{1-x^2}\equiv f\ ,\quad g=\frac{x^{\frac{1}{2}}y_2^{\frac{1}{2}}-x^{\frac{3}{2}}y_2^{-\frac{1}{2}}}{1-x^2}\left(y_1^{\frac{1}{2}}+y_1^{-\frac{1}{2}}\right)\ ,\quad \tilde{g}=0\ , \tag{3.2}$$

respectively.

Monopole operators in $U(N)\times U(N)$ come with fluxes $H=\{n_1\geq n_2\geq\cdots n_N\}$, $\tilde{H}=\{\tilde{n}_1\geq\tilde{n}_2\geq\cdots\geq\tilde{n}_N\}$ given by 2 sets of non-increasing integers. The way we calculate the index is by localization, closely following the analogous calculation for the ABJM theory [19]. See also [33, 34] for studies on the index with monopoles. In particular, we work in the path integral representation of the index, which refers to the Euclidean QFT on $S^2\times S^1$. We can deform the action of this theory by introducing the chiral part of the vector multiplet fields in $U(N)\times U(N)$, which is exact with our supercharge Q and has dimension 3 to preserve the scale invariance. The chiral part of the superpotential is also Q exact and can be turned off, leaving the anti-chiral part of the superpotential only in the action. The resulting path integral can be calculated exactly in the limit in which the former deformation dominates over the remaining parts of the action. One can easily check that the remaining anti-chiral part of the superpotential does not affect the calculation.[3] To explain the final result, it is convenient to

[3]Since the calculation appears insensitive to the detailed form of the superpotential, one may wonder if same calculations can be done in the $\mathcal{N}=2$ theory with either (2.8) or zero superpotential. It is not clear if (2.8) is at

introduce the following mode indices

$$f_m = \sum_{i,j=1}^{N} x^{|n_i - \tilde{n}_j|} f \left(e^{-i(\alpha_i - \tilde{\alpha}_j)} + e^{i(\alpha_i - \tilde{\alpha}_j)} \right) \tag{3.3}$$

$$f_{adj} = \sum_{i,j=1}^{N} x^{|n_i - n_j|} \left[g - (1 - \delta_{n_i n_j}) \right] e^{-i(\alpha_i - \alpha_j)} \ , \quad \tilde{f}_{adj} = - \sum_{i,j=1}^{N} x^{|\tilde{n}_i - \tilde{n}_j|} (1 - \delta_{\tilde{n}_i \tilde{n}_j}) e^{-i(\tilde{\alpha}_i - \tilde{\alpha}_j)},$$

where f_m is the contribution from bi-fundamental and anti-bifundamentals, f_{adj}, $\tilde{f}_{adj}$ that from the two adjoint matters and modes in the vector multiplets. The full index with given monopole charge H, $\tilde{H}$ is given by

$$\begin{aligned} I_{H,\tilde{H}} \quad = \quad & \frac{1}{(\text{symmetry})} \, x^{\frac{1}{2}\sum_{i,j=1}^{N} |n_i - \tilde{n}_j| - \sum_{i<j} |\tilde{n}_i - \tilde{n}_j|} \\ & \int_0^{2\pi} \prod_{i=1}^{N} \left[\frac{d\alpha_i d\tilde{\alpha}_i}{(2\pi)^2} \right] \prod_{i<j:\ n_i = n_j} \left[2 \sin \frac{\alpha_i - \alpha_j}{2} \right]^2 \prod_{i<j:\ \tilde{n}_i = \tilde{n}_j} \left[2 \sin \frac{\tilde{\alpha}_i - \tilde{\alpha}_j}{2} \right]^2 e^{ik \sum_{i=1}^{N} (n_i \alpha_i - \tilde{n}_i \tilde{\alpha}_i)} \\ & \exp \left[\sum_{n=1}^{\infty} \frac{1}{n} \left(f_m(x^n, y_1^n, y_2^n, n\alpha, n\tilde{\alpha}) + f_{adj}(x^n, n\alpha, n\tilde{\alpha}) + \tilde{f}_{adj}(x^n, n\alpha, n\tilde{\alpha}) \right) \right] \ . \end{aligned} \tag{3.4}$$

Se also [19] for detailed explanations on similar calculations. An important difference with the ABJM, apart from the different expressions for $f_m, f_{ad}, \tilde{f}_{adj}$, is the zero point energy on the first line. The first term comes from the bi-fundamental matters, and compared to ABJM there is a factor of $\frac{1}{2}$ since the number of chiral multiplets is reduced from 4 to 2. The second term comes from the vector multiplet in the second $U(N)$, which is the same as ABJM. In ABJM, there is another term $-\sum_{i<j} |n_i - n_j|$ coming from vector multiplet in first $U(N)$. In the dual ABJM, the two adjoint matters provide exactly the opposite contribution to cancel this.

In the large N limit keeping energy finite, we do the large N integration over distributions for holonomies $\alpha_i, \tilde{\alpha}_i$ which do not support magnetic flux. Just like the case studied in [19], the index then factorizes as

$$I_{free} I_> I_< \ , \tag{3.5}$$

as we explain now. The first part is the result of Gaussian integration,

$$I_{free} = \prod_{n=1}^{\infty} \frac{1}{1 - g - f^2} = \prod_{n=1}^{\infty} \frac{(1 - x^{2n})^2}{(1 - x^n y_2^{-n}) \left(1 - (xy_2)^{\frac{1}{2}} (y_1^{\frac{1}{2}} + y_1^{-\frac{1}{2}})(1 - x^2) - x^3 y_2 \right)} \tag{3.6}$$

a superconformal fixed point, or if it can flow to a fixed point of the type studied in [35]. If it does, and if that point is continuously connected with our $\mathcal{N}=3$ fixed point, the index from the former theory would be the same as ours. If it is not connected with $\mathcal{N}=3$ theory, we do not expect that the index computation followed in the main text applies for $k = 1$ case. This is because there are in general nontrivial quantum corrections in $\mathcal{N}=2$ case, especially for D-terms whose structure is crucial for the index computation and we do not have a proper understanding of these. This is in contrast with $\mathcal{N}=3$ where we have good control in D-terms and F-terms. The case with zero superpotential corresponds to an unstable fixed point. The fields here acquire anomalous dimensions [28] so that the Q exact deformation introduced in [19] could break dilatation symmetry.

and is exactly the part calculated in [32] without monopole operators. $I_>$, $I_<$, apart from the zero point energy part that we explain later, are given as follows. In terms of

$$\begin{aligned} \mathbf{f}^{m}_{ij} &= \left(x^{|n_i-\tilde{n}_j|} - x^{|n_i|+|\tilde{n}_j|}\right) f\left(e^{-i(\alpha_i-\tilde{\alpha}_j)} + e^{i(\alpha_i-\tilde{\alpha}_j)}\right) \\ \mathbf{f}^{adj}_{ij} &= \left[(g-1+\delta_{n_i n_j})x^{|n_i-n_j|} - (g-1)x^{|n_i|+|n_j|}\right] e^{-i(\alpha_i-\alpha_j)} \\ \tilde{\mathbf{f}}^{adj}_{ij} &= -\left[(1-\delta_{\tilde{n}_i\tilde{n}_j})x^{|\tilde{n}_i-\tilde{n}_j|} - x^{|\tilde{n}_i|+|\tilde{n}_j|}\right] e^{-i(\tilde{\alpha}_i-\tilde{\alpha}_j)} \ , \end{aligned} \tag{3.7}$$

$I_>$ is given by

$$\begin{aligned} &\frac{1}{(\text{symmetry})}\int_0^{2\pi}\prod_i\left[\frac{d\alpha_i}{2\pi}\right]\prod_j\left[\frac{d\tilde{\alpha}_j}{2\pi}\right]\prod_{i<j:\ n_i=n_j}\left[2\sin\frac{\alpha_i-\alpha_j}{2}\right]^2\prod_{i<j:\ \tilde{n}_i=\tilde{n}_j}\left[2\sin\frac{\tilde{\alpha}_i-\tilde{\alpha}_j}{2}\right]^2 e^{ik\sum_i(n_i\alpha_i-\tilde{n}_i\tilde{\alpha}_i)} \\ &\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}\left(\sum_{i,j}\mathbf{f}^{m}_{ij}(x^n,y_1^n,y_2^n,e^{in\alpha},e^{in\tilde{\alpha}}) + \sum_{i,j}\mathbf{f}^{adj}_{ij}(x^n,e^{in\alpha}) + \sum_{i,j}\tilde{\mathbf{f}}^{adj}_{ij}(x^n,e^{in\tilde{\alpha}})\right)\right] \end{aligned} \tag{3.8}$$

where $n_i, \tilde{n}_j$ are the positive part of the fluxes in $H, \tilde{H}$, and $\alpha, \tilde{\alpha}$ are associated holonomy. $I_<$ is given by a similar expression using negative fluxes in $H, \tilde{H}$ only. The whole integrand factorizes into $I_>$ and $I_<$ as shown above, as ABJM. The only remaining thing that one has to show is whether the zero point energy

$$x^{\frac{1}{2}\sum_{i,j=1}^{N}|n_i-\tilde{n}_j|-\sum_{i<j}|\tilde{n}_i-\tilde{n}_j|} \tag{3.9}$$

factorizes to two parts, where each part refers to positive or negative fluxes only.

$$\epsilon_0 = \frac{1}{2}\sum_{i,j=1}^{N}|n_i-\tilde{n}_j| - \sum_{i<j}|\tilde{n}_i-\tilde{n}_j| \ . \tag{3.10}$$

We now explain this factorization. Obviously there are many pairs which connect two positive or two negative fluxes, which factorize. We only have to consider the pairs connecting one positive and one negative flux. Its factorization is almost clear since $|x-y|=|x|+|y|$ if $x>0>y$. The only reason why we have to be careful is that the positive flux part may acquire constant coefficient depending on summation over $U(1)$'s with negative flux. To study this, let us define $N_\pm$ and $\tilde{N}_\pm$ to be the number of $U(1)$ Cartans which support positive/negative fluxes in two gauge groups. Then the contribution from pairs connecting positive and negative fluxes is

$$\begin{aligned} &\frac{1}{2}\Big((N-\tilde{N}_+)\sum|n_i^+| + (N-N_-)\sum|\tilde{n}_i^-| + (N-\tilde{N}_-)\sum|n_i^-| + (N-N_+)\sum|\tilde{n}_i^+|\Big) \\ &+(N-\tilde{N}_+)\sum|\tilde{n}_i^+| + (N-\tilde{N}_-)\sum|\tilde{n}_i^-| \ , \end{aligned} \tag{3.11}$$

where we included contributions from pairs with one zero flux and one nonzero flux. From this expression, the coefficients of the summations involving positive/negative fluxes only refer to the number of $U(1)$'s with positive/negative fluxes, respectively. This proves the factorization of zero point energy, and they should be included in the definition of $I_>$ and $I_<$. We also multiply $y_3^{\frac{h_1+h_2}{2}} \sim y_3^{\frac{k}{2}\sum_i n_i}$ to weight the operators with their $\frac{h_1+h_2}{2}$ charges.

Now we consider the case with $k=1$, which has the moduli space $\mathbb{C}^4$ and is thus suspected to be the dual description of ABJM. We can either compare the above index with that for the ABJM theory at $k=1$, or that over supersymmetric gravitons in $AdS_4\times S^7$ (which were checked to be the same [19]). Since there is a complete factorization $I_{free}I_>I_<$, it is straightforward to show that I_{free}, $I_>$ and $I_<$ should agree with indices from gravitons with zero KK-momenta, positive momenta, negative momenta separately [19]. Note also that, for $k=1$, the M-theory geometry is completely smooth so that the above supergravity index suffices. However, it has been shown in [32] that I_{free} does not agree with that from gravitons with zero KK-momenta. So whatever happens to $I_>$ and $I_<$ in the monopole sector, the two indices cannot agree.

In [23], the disagreement of the index in [32] was suspected to be due to the presence of an exotic branch in the moduli space if one analyzes the classical moduli space with (2.8). In the previous section, we re-analyzed the moduli space with the superpotential (2.7) at the $\mathcal{N}=3$ fixed point and showed the absence of this exotic branch. Therefore, the mismatch between the two indices should be somehow explained differently.[4]

At this point, we explain a feature of the index I_{free} which does not seem to be emphasized in [32]. The ranges of the chemical potentials x, y_1, y_2 should be such that the trace over the space of modes converges. This is guaranteed by taking x to be sufficiently small. Investigating the charges of all fields, one obtains the following conditions

$$xy_2^{\pm 1} < 1\ ,\quad x^3y_2^{\pm 1} < 1\ ,\quad xy_1^{\pm 1}y_2^{\pm 1} < 1\ ,\quad x^3y_1^{\pm 1}y_2^{\pm 1} < 1\ . \tag{3.12}$$

The equations involving x^3, coming from fermionic letters, are automatically implied once we accept the other conditions from bosons. With this understanding, let us review the calculation of large N saddle point calculation which led us to I_{free} in (3.6). The basic idea is that the integral over N holonomies reduces to a Gaussian integration over the distribution of them. To obtain (3.6), all the eigenvalues of the matrix appearing in the quadrature in the exponent should be positive. This is guaranteed for a sufficiently small x (which is analogous to the low temperature limit of the partition function). We should also check if some eigenvalues of this matrix can turn to be negative for some values of chemical potentials. Since (3.6) is nothing but the inverse of the determinant of this matrix, the sign change of any eigenvalue can be detected as the divergence of I_{free}. So let us have a look at the denominator of (3.6). The first factors involving $(1-(xy_2^{-1})^n)$ never approaches zero in the allowed range (3.12). However, the second factor

$$\left(1-(xy_2)^{\frac{1}{2}}(y_1^{\frac{1}{2}}+y_1^{-\frac{1}{2}})(1-x^2)-x^3y_2\right) \tag{3.13}$$

can approach zero in the allowed range (3.12), as follows. Firstly, it is much simpler to consider the simple case with $y_1=y_2=1$, in which case the allowed region is simply $x<1$. Then the

[4] Of course one explanation would simply be regarding this as the evidence that dual ABJM at $k=1$ does not describe M2-branes in $\mathbb{R}^8$.

above factor becomes

$$1-2x^{\frac{1}{2}}+2x^{\frac{5}{2}}-x^3=(1-x)\left(1-2x^{\frac{1}{2}}+x-2x^{\frac{3}{2}}+x^2\right)\ . \tag{3.14}$$

The expression appearing in the second parenthesis monotonically decreases from 1 to -1 in the range $0<x<1$, which becomes zero approximately at $x^{\frac{1}{2}}\approx 0.5310$. Beyond this value, one eigenvalue becomes negative and the previous saddle point at which the distribution function is uniform seizes to be a minima. Instead one should find a new saddle point at which the distribution function is not uniform [36]. The index undergoes a transition to a 'deconfined' phase.

At the above deconfined phase, the (index version of) free energy is proportional to N^2 [36], which is much larger than $N^{\frac{3}{2}}$ which we expect in the high temperature phase of M2 branes. We should emphasize that, in all Chern-Simons-matter type gauge theory models for M2 branes, the N^2 weakly coupled degrees of freedom should not all appear in the strongly interacting regime (say at $k=1$) for these theories to describe M2 branes. Actually the index never deconfines in the case of ABJM [19]. The fact that N^2 degrees of freedom appear in the index for the 'dual ABJM' seems to be in sharp contrast with the M2 brane picture.

Acknowledgments

The work of S.K. is supported in part by the Research Settlement Fund for the new faculty of Seoul National University. The work of J.P. is supported in part by KOSEF Grant R01-2008-000-20370-0, by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MEST) with the grant number 2009-0085995 and by the grant number 2005-0049409 through the Center for Quantum Spacetime(CQUeST) of Sogang University.

References

[1] J. H. Schwarz, "Superconformal Chern-Simons theories," JHEP **0411** (2004) 078 [arXiv:hep-th/0411077].

[2] J. Bagger and N. Lambert, "Modeling multiple M2's," Phys. Rev. D **75** (2007) 045020 [arXiv:hep-th/0611108].

[3] J. Bagger and N. Lambert, "Gauge symmetry and supersymmetry of multiple M2-branes," Phys. Rev. D **77** (2008) 065008 [arXiv:0711.0955 [hep-th]].

[4] J. Bagger and N. Lambert, "Comments on multiple M2-branes," JHEP **0802** (2008) 105 [arXiv:0712.3738 [hep-th]].

[5] A. Gustavsson, "Algebraic structures on parallel M2-branes," Nucl. Phys. B **811**, 66 (2009) [arXiv:0709.1260 [hep-th]] .

[6] A. Gustavsson, "Selfdual strings and loop space Nahm equations," JHEP **0804**, 083 (2008) [arXiv:0802.3456 [hep-th]].

[7] M. Van Raamsdonk, "Comments on the Bagger-Lambert theory and multiple M2-branes," JHEP **0805**, 105 (2008) [arXiv:0803.3803 [hep-th]].

[8] K. Hosomichi, K. M. Lee, S. Lee, S. Lee and J. Park, "N=4 Superconformal Chern-Simons theories with hyper and twisted hyper multiplets," JHEP **0807**, 091 (2008) [arXiv:0805.3662 [hep-th]].

[9] D. Gaiotto and E. Witten, "Janus configurations, Chern-Simons couplings, and the theta-angle in N=4 Super Yang-Mills theory," arXiv:0804.2907 [hep-th].

[10] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, "N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals," JHEP **0810**, 091 (2008) [arXiv:0806.1218 [hep-th]].

[11] K. Hosomichi, K. Lee , S. Lee , S. Lee and J. Park, "N=5,6 Superconformal Chern-Simons Theories and M2-branes on Orbifolds," JHEP **0809** 002 (2008)[arXiv:0806.4977 [hep-th]].

[12] M. Bandres, A. Lipstein and J. Schwarz, "Studies of the ABJM Theory in a Formulation with Manifest SU(4) R-Symmetry," JHEP **0809** 027 (2008)[arXiv:0807.0880 [hep-th]].

[13] D. Martelli and J. Sparks, " Moduli spaces of Chern-Simons quiver gauge theories and AdS(4)/CFT(3)," Phys. Rev. D **78**, 126005 (2008) [arXiv:0808.0912 [hep-th]].

[14] A. Hanany and A. Zaffaroni, " Tilings, Chern-Simons theories and M2 branes," JHEP **0810** (2008) 111 [arXiv:0808.1244 [hep-th]].

[15] F. Benini, C. Closset and S. Cremonesi, "Chiral flavors and M2-branes at toric CY4 singularities," JHEP **1002** 036 (2010) [arXiv:0911.4127 [hep-th]].

[16] N. Kim, "Quiver Chern-Simons theories and 3-algebra orbifolds," [arXiv:0912.5418 [hep-th]].

[17] N. Benishti , Yang-Hui He and J. Sparks, "(Un)Higgsing the M2-brane," **JHEP** 1001 (2010) 067, [arXiv:0909.4557 [hep-th]].

[18] J. Bhattacharya and S. Minwalla, "Superconformal indices for $\mathcal{N} = 6$ Chern-Simons theories," JHEP **0901**, 014 (2009) [arXiv:0806.3251 [hep-th]].

[19] S. Kim, "The complete superconformal index for N=6 Chern-Simons theory," Nucl. Phys. B **821**, 241 (2009) [arXiv:0903.4172 [hep-th]].

[20] A. Kapustin, B. Willett and I. Yaakov, "Exact results for Wilson loops in superconformal Chern-Simons theories with matter," arXiv:0909.4559 [hep-th].

[21] N. Drukker and D. Trancanelli, "A supermatrix model for N=6 super Chern-Simons-matter theory," JHEP **1002**, 058 (2010) [arXiv:0912.3006 [hep-th]].

[22] M. Marino and P. Putrov, "Exact results in ABJM Theory from topological strings," arXiv:0912.3074 [hep-th].

[23] D. Rodriguez-Gomez, "M5 spikes and operators in the HVZ membrane theory," JHEP **1003**, 039 (2010) [arXiv:0911.0008 [hep-th]].

[24] O. Aharony , O. Bergman and D. Jafferis, " Fractional M2-branes," JHEP **0811** 043 (2008). [arXiv:0807.4924 [hep-th]].

[25] S. Lee, "Superconformal field theories from crystal lattices," Phys.Rev.D **75** 101901 (2007) [arXiv:hep-th/0610204].

[26] S. Lee, S. Lee and J.Park, "Toric AdS4/CFT3 duals and M-theory Crystals," JHEP **0705** 004 (2007) [arXiv: hep-th/0702120].

[27] S. Kim , S. Lee, S. Lee and J. Park, "Abelian Gauge Theory on M2-brane and Toric Duality," Nucl.Phys.B **797** 340 (2008) [arXiv:0705.3540 [hep-th]].

[28] D. Gaiotto and X. Yin, "Notes on superconformal Chern-Simons-matter theories," JHEP **0708**, 056 (2007) [arXiv:0704.3740 [hep-th]].

[29] A. Hanany , D. Vegh and A. Zaffaroni, "Brane tilings and M2 branes," JHEP **0903** (2009) 012, [arXiv:0809.1440 [hep-th]].

[30] S. Franco , A. Hanany , J. Park and D. Rodriguez-Gomez, "Towards M2-brane theories for generic toric singularities," JHEP **0812** (2008) 110, [arXiv:0809.3237 [hep-th]].

[31] N. Lambert and D. Tong, "Membranes on an Orbifold," Phys. Rev. Lett. **101**, 041602 (2008) [arXiv:0804.1114 [hep-th]].

[32] J. Choi, S. Lee and J. Song, "Superconformal indices for orbifold Chern-Simons theories," JHEP **0903**, 099 (2009) [arXiv:0811.2855 [hep-th]].

[33] S. Kim and K. Madhu, "Aspects of monopole operators in N=6 Chern-Simons theory," JHEP **0912**, 018 (2009) [arXiv:0906.4751 [hep-th]].

[34] Y. Imamura and S. Yokoyama, "A monopole index for N=4 Chern-Simons theories," Nucl. Phys. B **827**, 183 (2010) [arXiv:0908.0988 [hep-th]].

[35] C. M. Chang and X. Yin, "Families of conformal fixed points of N=2 Chern-Simons-matter theories," arXiv:1002.0568 [hep-th].

[36] O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas and M. Van Raamsdonk, "The Hagedorn / deconfinement phase transition in weakly coupled large N gauge theories," Adv. Theor. Math. Phys. **8**, 603 (2004) [arXiv:hep-th/0310285].